DOI: https://doi.org/10.30837/MMP.2022.016

DECISION MAKING SUPPORT UNDER CONDITIONS OF INCOMPLETE CONSISTENCY OF EXPERT ADVANTAGES

Beskorovainyi V., Kolesnyk L., Russkin V.

To expand the prospects of intellectualization of design and management procedures for complex objects, a decision of an urgent scientific and applied problem of increasing the efficiency of multicriteria decision support technologies is proposed. A combined method for evaluating variants under conditions of incomplete consistency of expert advantages is proposed. To assess the importance of partial criteria it was proposed to use methods of reducing variance and penalizing inconsistency, allowing to increase the accuracy of assessments of weight coefficients of partial criteria. With low accuracy in determining the weighting coefficients to assess the effectiveness of decisions it is proposed to use a universal common utility function, which by changing one of the parameters allows to implement strategies for finding both the most effective and the most sustainable decisions. Practical use of the proposed method will allow to obtain more effective decisions to multi-criteria optimization problems by increasing the accuracy of assessment of weight coefficients of partial criteria. The direction of further research may be the development of effective decision support methods for fuzzy or interval characteristics of variants.

Introduction

Decision-making in project management, computer-aided design or control systems is usually carried out taking into account many heterogeneous indicators and constraints under conditions of incomplete definition of goals and input data. Typical examples are decision-making tasks in the development and implementation of investment projects, conducting tenders, product certification, choosing suppliers or contractors, optimization of variants for the construction of objects, etc. At the same time, the decisions made in such conditions, in particular, must be reasoned, objective, reproducible and protected from the authoritarian influence of individuals or organizations. It is known that the choice of decisions on a set of heterogeneous contradictory indicators, even for clearly defined goals and inputs, is a rather difficult problem [1–3].

The central task of this problem is to synthesize an adequate mathematical model for forming a scalar multifactor assessment of the effectiveness of decisions from the set of admissible $x \in X$. The complexity of the problem lies in the fact that a set of partial criteria is used to evaluate the decision variants $k_i(x)$, $i = \overline{1,n}$ each of which has its functional meaning, dimensionality, interval, and direction of the desired change improvement. In such cases, to rank the decisions and choose the best among them on the set of admissible $D(m,\delta)$ is carried out based on the utility maximization paradigm [4]. The decision maker (DMP) carries out the ordering of a small set of variants $D\left(N, m_0, \delta_0\right)$ in the framework of the ordinalistic approach. The cardinalistic approach to problem solving involves the formation of a generalized criterion of effectiveness $J_E\left(c\right) = \sup\left\{\left|v\left(k, \overline{w}, c\right)\right| : k \in N, \overline{w} \in E\right\}$, using which the generalized evaluation and selection of the best variant is carried out:

$$x^o = \arg\max_{x \in X} P\left(x\right). \tag{1}$$

The value of the generalized criterion of effectiveness $\overline{w}$ (1) allows organizing variants by value $\forall x, y \in X$:

$$x \sim y \leftrightarrow P\left(x\right) \ = \ P\left(y\right);\ x > y \leftrightarrow P\left(x\right) \ > \ P\left(y\right);\ x \geq y \leftrightarrow P\left(x\right) \geq P\left(y\right). \tag{2}$$

In many cases, the processes of design, development planning and reengineering of complex objects involve the generation and analysis in automatic mode of super powered sets of alternative decisions, most of which are inefficient [5]. The final choice of the best decision is made by an RRO capable of analyzing a relatively small number of variants. The above leads to the need for clear coordination between automatic and expert DMP procedures (6). In addition, expert evaluations regarding the importance of individual indicators can vary significantly, and the partial criteria $k_i\left(x\right)$, $i = \overline{1, n}$, characterizing decision variants can be set not by their point values, but as fuzzy sets [7], random values distributed according to some law, or intervals in which the benefits are not set [4, 7–8]. On this basis, to extend the prospects of intellectualization of design and management procedures for complex objects, an urgent scientific and applied problem is the development of technologies to support collective decision-making under conditions of multicriteria and incomplete consistency of expert advantages.

The object of the study are processes of support for making design and management decisions in conditions of incomplete consistency of expert advantages.

The subject of the study are methods to support collective multi-criteria design and management decisions in conditions of incomplete consistency of expert advantages.

The aim of the work is to improve the effectiveness of decision support technologies by developing a combined method for evaluating variants in conditions of incomplete consistency of expert advantages.

Multi-criteria model of the collective decision-making problem

In the early stages of formalization, the decision-making task in terms of goals, means, and results is presented in such a form [4]:

$$\Phi : X \times S \to Z, \tag{3}$$

where X – set of alternatives; S – set of environmental states, characterizing the manifestation of uncertainty in the decision-making process; Z – set of consequences (results of the decision-making problem); Φ – some mapping.

In notations (3), the decision-making process consists in choosing a subset of alternatives from the set X according to some principle of optimality P (2), and the decision-making problem is to choose alternatives $x \in X$, which leads to some result $z \in Z$ under the state of the environment $s \in S$ [4].

Efficiency of problem decision $x \in X$ is determined by the degree of correspondence of the obtained result $z \in Z$ to the set goals, evaluated by values of the set of chosen partial criteria $k_i(x)$, $i = \overline{1,n}.$, quantitative characteristic of efficiency of each alternative $x \in X$ is utility function $P(x)$, depending on values of which the choice of decision $x^o \in X$ is made (1). The process of choice $x^o \in X$ is called a decision-making procedure, and the result of choice x^o is the best (optimal, effective) decision.

The problem of collective decision-making is considered in this formulation. Given: a set of alternatives $X = \{x\}$, each of which is characterized by a set of partial criteria $\{k_i(x)\}$, $i = \overline{1,n}.$ It is necessary to determine the best alternative from the set of admissible $x^o \in X$, if the significance of partial criteria $\lambda = [\lambda_i]$, $\sum_{i=1}^{n} \lambda_i = 1$, $\lambda_i \geq 0$, $i = \overline{1,n}$, determined by a group of experts, and the estimates of each of the experts $\lambda' = \left[\lambda_i'^{\,j}\right]$, $j = \overline{1,m}$ significantly different.

The most common for assessing the generalized utility of decision variants $P(x)$ is an additive function of the type:

$$P(x) = \sum_{i=1}^{n} \lambda_i \xi_i(x), \tag{4}$$

where $\xi_i(x)$ – value of the partial criterion utility function $k_i(x), 0 \leq \xi_i(x) \leq 1$, $i = \overline{1,n}$ for the decision x.

The minimum number of machine operations to calculate their values among the common requires a partial criterion utility function with a parameter value $\alpha_i = 1$ [4]:

$$\xi_i(x) = \left\{ \left[k_i(x) - k_i^- \right] \Big/ \left[k_i^+ - k_i^- \right] \right\}^{\alpha_i}, \quad i = \overline{1,n}, \tag{5}$$

where $k_i(x)$, k_i^+, k_i^- – the value of the i-th partial criterion for the decision x, its best and worst value.

Function (5) is monotonous and dimensionless, has a single interval of variation from 0 to 1, is invariant to the form of extremum of partial criterion, allows to realize both linear and non-linear (convex upward and downward) dependence on values of partial criterion. For more precise *S*- and *Z*- like approximation of the estimates of the partial criteria values is proposed to use a universal glue function, which is the best in terms of the complex indicator «accuracy-complexity» [9–10]:

$$\xi(x) = \begin{cases} \bar{a}(b_1+1)\left(1 - \left(b_1 \Big/ \left(b_1 + \dfrac{\bar{k}(x)}{\bar{k}_a}\right)\right)\right), 0 \le \bar{k}(x) \le \bar{k}_a; \\ \bar{a} + (1-\bar{a})(b_2+1) \times \left(1 - \left(b_2 \Big/ \left(b_2 + \dfrac{\bar{k}(x) - \bar{k}_a}{1 - \bar{k}_a}\right)\right)\right), \bar{k}_a < \bar{k}(x) \le 1, \end{cases} \tag{6}$$

where $\bar{k}(x)$ – the value of the partial criteria utility function (5) for $\alpha_i = 1$; $\bar{k}_a, \bar{a}$ – normalized coordinate values of the glue point of the function, $0 \le \bar{k}_a \le 1$, $0 \le \bar{a} \le 1$; b_1, b_2 – parameters that determine the type of dependence on the initial and final segments of the function.

The most adequate for assessing the generalized utility of decision variants $P(x)$ is a function based on the Kolmogorov–Gabor polynomial [4–6]:

$$P(x) = \sum_{i=1}^{n} \lambda_i \xi_i(x) + \sum_{i=1}^{n} \sum_{j=i}^{n} \lambda_{ij} \xi_i(x) \xi_j(x) + \sum_{i=1}^{n} \sum_{j=i}^{n} \sum_{l=j}^{n} \lambda_{ijl} \xi_i(x) \xi_j(x) \xi_l(x) + \ldots, \tag{7}$$

where λ_i, λ_{ij}, λ_{ijl} – coefficients of importance of the criteria $k_i(s)$, $i = \overline{1,n}$ and criterion products $k_i(x)$, $k_j(x)$, $k_l(x)$.

The generalized utility function (4) is a special case of function (7). By entering a set of notations $\lambda_{1,2} = \lambda_{n+1}$, $\lambda_{1,3} = \lambda_{n+2}, \ldots$, $\xi_1(x)\xi_2(x) = \xi_{n+1}(x)$, $\xi_1(x)\xi_3(x) = \xi_{n+2}(x), \ldots$ function (7) can be represented in the form (4) at $n = N$ (where N – total number of addends in the function (7)).

In the technologies of collective decision-making to solve the problem (1) using models (4) – (7) it is necessary to set estimates of weight coefficients of partial criteria $\lambda = [\lambda_i]$, $i = \overline{1,n}$ based on contradictory expert opinions $\lambda' = \left[\lambda_i'^{\,j} \right]$, $j = \overline{1,m}$ (where $\lambda_i'^{\,j}$ – the significance of the i-th partial criterion is determined by the j-th expert; m – the number of experts).

Parametric synthesis of a collective decision-making model

If the estimates of the experts $\lambda' = \left[\lambda_i'^{\,j} \right]$, $i = \overline{1,n}$, $j = \overline{1,m}$ regarding the importance of the selected partial criteria $k_i(x)$ relatively consistent, then the best decision is chosen using model (4) or (7) for their generalized values $\lambda_i = \lambda_i^*$, $i = \overline{1,n}$, relatively consistent, then the best decision is chosen using model (4) or (7) for their generalized values

$$\lambda_i = \lambda_i^* = \bar{\lambda}_i = \frac{1}{m} \sum_{j=1}^{m} \lambda_i'^{\,j}, \; i = \overline{1,n}. \tag{8}$$

If the experts' evaluations turn out to be insufficiently consistent, a re-examination is performed. If it is impossible or inexpedient to conduct a repeated examination, special methods of evaluating the importance of partial criteria are used: reduction of variance, penalization of inconsistency, etc.

In the method of variance reduction (method 3) for ordered series of expert evaluations of weighting coefficients λ_i^j, $i = \overline{1,n}$, $j = \overline{1,m}$ the positions of the lower and upper quartiles are determined. The scores that fall into them are not taken into account when determining the average values $\lambda_i = \lambda_i^* = \bar{\lambda}_i$.

In [11] it is hypothesized that the criteria for which the DMP It is proposed to consider that such criteria are more reliable for the construction of the decision-making procedure compared to the criteria for which the collective opinion of the DMP has a higher degree of uncertainty. Therefore, it is proposed to penalize the inconsistency of experts' advantages by reducing the values of the corresponding weight coefficients (method 4).

The algorithm of this method for determining $\lambda_i = \lambda_i^*$, $i = \overline{1,n}$ assumes the implementation of the following steps [11]:

– determination of the arithmetic mean values of expert evaluations for all weight coefficients:

$$\bar{\lambda}_i = \frac{1}{m} \sum_{j=1}^{m} \lambda_i'^{\,j}, \; i = \overline{1,n}; \tag{9}$$

– determination of the entropy of values:

$$E_i = \sqrt{\pi/2} \times \frac{1}{m} \sum_{j=1}^{m} \left| \lambda_i'^{\,j} - \bar{\lambda}_i \right|, \; i = \overline{1,n}; \tag{10}$$

– determination of the hyperentropy of values:

$$H_i = \sqrt{S_i^{\,2} - E_i^{\,2}}, \; i = \overline{1,n}, \tag{11}$$

where $S_i^{\,2}$ – dispersion of the i-th weighting coefficient [12]:

$$S_i^{\,2} = \frac{1}{m} \sum_{j=1}^{m} \left(\lambda_i'^{\,j} - \bar{\lambda}_i \right)^2; \tag{12}$$

– the ratio [11] is used to determine the estimates $\lambda_i = \lambda_i^*$:

$$\begin{cases} \lambda_i^* = \arg\min \max\limits_{ij} \left\{ \left| H_i \lambda_i^* - H_i \lambda_i^j \right| \right\}, \\ \sum\limits_{i=1}^{n} \lambda_i^* = 1, \; \lambda_i^* \geq 0 \;\; \forall i = \overline{1,n}, \; \forall j = \overline{1,m}. \end{cases} \tag{13}$$

The optimization model of the problem (13) is transformed into a linear programming problem using the following relations:

$$\begin{cases} \min \zeta, \\ H_i \lambda_i^* - H_i \lambda_i^j \leq \zeta, \; H_i \lambda_i^* - H_i \lambda_i^j \geq -\zeta, \\ \sum\limits_{i=1}^{n} \lambda_i^* = 1, \; \lambda_i^* \geq 0 \;\; \forall i = \overline{1,n}, \; \forall j = \overline{1,m}. \end{cases} \tag{14}$$

In this case, the best values of the weight coefficients of the partial criteria $\lambda_i = \lambda_i^*$, $i = \overline{1,n}$ are the decisions to the problem (14).

If the accuracy of determining the weighting coefficients is low, it is recommended to use a universal model (method 5) to evaluate the effectiveness of decisions [4]:

$$P(x) = \left\{ \frac{1}{n} \sum_{i=1}^{n} \left[\lambda_i \, k_i(x) \right]^{\beta} \right\}^{1/\beta}, \tag{15}$$

where β – a parameter that depends on the error in determining (scatter) the weighting coefficients, and determines the trade-off scheme for selecting the best decision.

At $\beta = 1$ model (15) allows us to choose the decisions that have the maximum additive utility (4), and at $|\beta| \to \infty$ implement a maximin or minimax scheme to choose compromise decisions:

$$x^o = \arg\max_{x \in X} \sum_{i=1}^{n} \lambda_i \xi_i(x), \tag{16}$$

$$x^o = \arg\max_{x \in X} \min_{1 \le i \le n} \{\lambda_i \, \xi_i(x)\}, \tag{17}$$

$$x^o = \arg\min_{x \in X} \max_{1 \le i \le n} \{\lambda_i \, \xi_i(x)\}, \tag{18}$$

where X – is the set of admissible decisions.

If the weighting coefficients are defined with an error $\delta = \max_{ij} \left| \lambda_i^j - \overline{\lambda}_i \right|$, the corresponding value of the parameter is determined by the relation:

$$\beta = \log n / \log(1 + \delta). \tag{19}$$

To establish the effectiveness of methods for determining the weighting coefficients of share criteria in models of multicriteria decision-making, let us perform their experimental study.

Results of experiments

Consider the problem of parametric synthesis of model (4) and choice, using it, of the best decision from the set of admissible $X = \{x_l\}$, $l = \overline{1,8}$, evaluated by three indicators $k_i(x)$, $i = \overline{1,3}$.

To simplify the perception of the conditions of the problem and the results of the decision variants from the set of admissible decisions are presented using the utility functions of partial criteria $\xi_i(x_l)$, $i = \overline{1,3}$ (5) with a value of the parameter $\alpha_i = 1$ (table 1).

The normalized results of the range $[0;1]$ of partial criteria λ_i^j, $i = \overline{1,3}$ significance evaluation by the experts E_j, $j = \overline{1,10}$ are shown in table 2.

Let us determine the generalized values of the importance of partial criteria $\lambda_i = \lambda_i^*$, $i = \overline{1,n}$ by averaging expert evaluations (method 1) (8), calculating medians (method 2), reducing dispersion (method 3), and penalizing inconsistency of expert advantages (method 4).

To perform the constraints of the task $\sum_{i=1}^{n} \lambda_i = 1$, $\lambda_i \ge 0$, $i = \overline{1,n}$ let us normalize the obtained estimates by the ratio $\lambda_i = \frac{1}{n} \sum_{i=1}^{n} \lambda_i^*$, $i = \overline{1,n}$ (table 3).

Table 1

Characteristics of the decision variants

x_l \ $\xi_i(x_l)$	$\xi_1(x_l)$	$\xi_2(x_l)$	$\xi_3(x_l)$
x_1	0,9486	0,5840	0,5066
x_2	0,0172	0,9545	0,1645
x_3	0,8194	0,9827	0,0135
x_4	0,2875	0,6911	0,9773
x_5	0,3086	0,3289	0,6844
x_6	0,4463	0,0157	0,7646
x_7	0,9873	0,5088	0,5464
x_8	0,6104	0,5962	0,5965

Table 2

Results of the criteria significance evaluation by experts

E_j \ $k_i(x_l)$	$k_1(x_l)$	$k_2(x_l)$	$k_3(x_l)$
E_1	0,1	0,7	0,2
E_2	0,2	0,6	0,2
E_3	0,3	0,6	0,1
E_4	0,3	0,6	0,1
E_5	0,3	0,5	0,2
E_6	0,2	0,7	0,1
E_7	0,1	0,7	0,2
E_8	0,3	0,5	0,2
E_9	0,2	0,7	0,1
E_{10}	0,2	0,6	0,2

Table 3

Normalized values of the criterion importance estimates

Method \ λ_i	λ_1	λ_2	λ_3
Method 1	0,220	0,620	0,160
Method 2	0,200	0,600	0,200
Method 3	0,226	0,613	0,161
Method 4	0,212	0,596	0,192
Method 5	0,220	0,620	0,160

For the normalized values of the weight coefficients λ_i, $i=\overline{1,n}$ (table 3) using the additive model (4) by methods 1–4 and for the value $\beta=-9,6940$, which corresponds to the error (dispersion) $\delta=0,12$ by method 5 (15) calculate estimates of the generalized utility of decision variants $P(x_l)$ (table 4).

Table 4

Estimates of the overall utility of the variants

$P(x_l)$	Method 1	Method 2	Method 3	Method 4	Method 5
$P(x_1)$	0,6518	0,6414	0,6538	0,6463	0,3234
$P(x_2)$	0,6219	0,6090	0,6154	0,6041	0,5284
$P(x_3)$	0,7917	0,7562	0,7895	0,7619	0,5439
$P(x_4)$	0,6481	0,6676	0,6461	0,6606	0,3826
$P(x_5)$	0,3813	0,3959	0,3817	0,3929	0,1821
$P(x_6)$	0,2302	0,2516	0,2337	0,2508	0,1105
$P(x_7)$	0,6200	0,6120	0,6229	0,6174	0,2824
$P(x_8)$	0,5994	0,5991	0,5995	0,5992	0,3300

According to the obtained values of the total utility of the variants (Table 4) let us determine on the set $X=\{x_l\}$, $l=\overline{1,8}$ (Table 1) ratio of strict advantage:

$$R_S(X)=\left\{<x_j,x_l>:\ x_j,\ x_l\in X,\ x_j>x_l\right\}. \tag{20}$$

Based on the relations of strict advantage (20) for the set of variants (table 1) the corresponding orders are constructed (table 5).

Table 5

Ordering decisions by utility

Methods	Orders
Method 1	$x_3 \succ x_1 \succ x_4 \succ x_2 \succ x_7 \succ x_8 \succ x_5 \succ x_6$
Method 2	$x_3 \succ x_4 \succ x_1 \succ x_7 \succ x_2 \succ x_8 \succ x_5 \succ x_6$
Method 3	$x_3 \succ x_1 \succ x_4 \succ x_7 \succ x_2 \succ x_8 \succ x_5 \succ x_6$
Method 4	$x_3 \succ x_4 \succ x_1 \succ x_7 \succ x_2 \succ x_8 \succ x_5 \succ x_6$
Method 5	$x_3 \succ x_2 \succ x_4 \succ x_8 \succ x_1 \succ x_7 \succ x_5 \succ x_6$

To clarify the degree of consistency of the order relations of the alternatives, we use the Spearman coefficient (Table 6).

The obtained values of Spearman coefficients for methods 1–4, by means of which the weight coefficients of partial criteria were determined, are close to the one, which indicates small discrepancies in the indicated orders. This is a consequence of the relatively insignificant divergence of the experts' assessments (Table 2).

Table 6

Spearman coefficient

Methods	Method 1	Method 2	Method 3
Method 2	0,952380952		
Method 3	0,976190476	0,976190476	
Method 4	0,952380952	1	0,976190476

Some difference between the evaluations of the total utility of variants and the corresponding order for method 5 (15) and those obtained by methods 1–4 can be explained by its focus on the maximizing scheme of decision-making (17). It presupposes a possible change in the weighting coefficients in the range of $\lambda_i \pm \delta$, $i = \overline{1,n}$ and takes into account the minimum value of the utility function of partial criteria for each of the variants.

Conclusions

The analysis of the current state of the problem of support for multi-criteria design and management decisions found that in many cases, expert assessments regarding the importance of individual indicators can vary significantly, and the individual criteria characterizing the decision variants can be set not by their point values, but in the form of fuzzy sets. The following are the most important criteria of a decision, which can be defined not as their point values, but as fuzzy sets of random values distributed according to some law, or intervals, in which the advantages are not defined.

On this basis, to expand the prospects of intellectualization of design and management procedures for complex objects, the decision of urgent scientific and applied problem of increasing the efficiency of multicriteria decision support technologies by developing a combined method for evaluating variants in conditions of incomplete consistency of expert benefits is proposed. When it is impossible or inexpedient to carry out re-examination to assess the importance of partial criteria, it is proposed to use methods to reduce variance and penalize inconsistency, allowing to increase the accuracy of assessments of the weight coefficients of partial criteria. In cases of low accuracy in determining the weight coefficients of assessment of the effectiveness of decisions it is proposed to use the universal function of total utility, which by changing one of the parameters allows to implement strategies for finding both the most effective and the most sustainable decisions.

Practical use of the proposed method will allow by increasing the accuracy of assessment of weight coefficients of partial criteria to obtain more effective decisions to problems of multi-criteria optimization. The direction of further research may be the development of effective decision support methods for fuzzy or interval characteristics of variants.

References

1. Fakhrehosseini, S. F. (2020), «Selecting the optimal industrial investment by multi-criteria decision-making methods with emphasis on TOPSIS, VIKOR and COPRAS (Case Study of Guilan Province)», *International Journal of Research in Industrial Engineering,* Vol. 8(4), P. 312–324. **DOI**: https://doi.org/10.22105/riej.2020.216548.1117
2. Zlaugotne, B., Zihare, L., Balode, L., Kalnbalkite, A., Khabdullin, A., Blumberga, D. (2020), «Multi-Criteria Decision Analysis Methods Comparison», *Environmental and Climate Technologies*, No. 24, Issue 1, P. 454–471. **DOI**: 10.2478/rtuect-2020-0028
3. Greco, S., Ehrgott, M., Figueira, J. R. (2016), *«Multiple criteria decision analysis. State of the Art Surveys»,* Publ. Springer, New York, 1346 p.
4. Ovezgeldiev, A.O., Petrov, E.G., Petrov, K.E. (2002), «*Synthesis and identification of multifactorial models for estimation and optimization*», Kyiv: Naukova dumka, 164 p.
5. Beskorovainyi, Vladimir V., Petryshyn, Lubomyr B., Shevchenko, Olha Yu. (2020), «Specific subset effective option in technology design decisions», *Applied Aspects of Information Technology,* Vol. 3, No. 1, P. 443-455. **DOI**: https://doi.org/10.15276/aait.05.2022.1
6. Beskorovainyi, V. (2020), «Combined method of ranking options in project decision support systems», *Innovative Technologies and Scientific Solutions for Industries*, No. 4 (14), P. 13–20. **DOI**: https://doi.org/10.30837/ITSSI.2020.14.013
7. Huynh, V. N., Nakamori, Y., Ryoke, Ho, T. B. (2007), «Decision making under uncertainty with fuzzy targets», *Fuzzy Optimization and Decision Making,* Vol. 6, P. 255–278. **DOI**: https://doi.org/10.1007/s10700-007-9011-0
8. Petrov, E., Brynza, N., Kolesnyk, L., Pisklakova, O. (2014), «*Methods and models of decision-making under conditions of multi-criteria and uncertainty*», Herson: Grin D.S., 192 p.
9. Beskorovainyi V., Berezovskyi G. Estimating the properties of technological systems based on fuzzy sets // Innovative technologies and scientific solutions for industries. 2017. № 1 (1). C. 14–20. **DOI:** https://doi.org/10.30837/2522-9818.2017.1.014
10. Beskorovainyi V., Berezovskyi H. Identification of preferences in decision support systems // ECONTECHMOD. 2017. Vol. 06. №4. P. 15–20.
11. Khorshidi, H, Aickelin, U. (2020), «Multicriteria Group Decision-Making Under Uncertainty Using Interval Data and Cloud Models», *Journal of the Operational Research Society,* Vol. 72, Issue 11, P. 2542–2556. **DOI**: https://doi.org/10.1080/01605682.2020.1796541
12. Kovaleva, M., Voloshin, S. (2019), «*Data analysis*», Moskow: Mir nauki, 129 p.
13. Guerra, M. L., Stefanini, L. (2012), «A comparison index for interval ordering based on generalized Hukuhara difference», *Soft Computing,* Vol. 16 (11), P. 1–25. **DOI**: https://doi.org/10.1007/s00500-012-0866-9
14. Moore, R. E., Kearfott, R. B., Cloud M.J. (2009), «*Introduction to interval analysis*», Philadelphia: Society for Industrial and Applied Mathematics, 213 p. **DOI**: https://doi.org/10.1137/1.9780898717716
15. Kosheleva, O., Kreinovich, V., Pham, U. (2021), «Decision-making under interval uncertainty revisited», *Asian Journal of Economics and Banking*, Vol. 5 (1), P. 79–85. **DOI**: https://doi.org/10.1108/AJEB-07-2020-0030
16. Stefanini, L., Guerra, M. L., Amicizia B. (2019), «Interval Analysis and Calculus for Interval-Valued Functions of a Single Variable. Part I: Partial Orders, gH-Derivative, Monotonicity», *Axioms*, Vol. 8 (113), P. 1–30. **DOI**: https://doi.org/10.3390/axioms8040113